

# Context-Adaptive Statistical Inference: Recent Progress, Open Problems, and Opportunities for Foundation Models

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# Abstract

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Context-adaptive inference enhances statistical methods by allowing model parameters to shift with context, either explicitly through parameter learning or implicitly via interactions between context and input features. In this review, we outline recent progress in integrating context into statistical models and explore the potential of foundation models to serve as context providers. We conclude by discussing future trends, challenges, and opportunities in context-adaptive statistical inference.

## Introduction

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### Purpose and Scope

TODO: Establishing the framework for our examination of context-adaptive statistical methods and the significance of foundation models.

### Conceptual Foundations

Unpacking the core principles and historical impact of adaptive methods within statistical modeling.

### A Brief History of Personalized Inference

Personalization aims to solve the problem of *parameter heterogeneity*, where model parameters are *sample-specific*.

$$X_i \sim P(X; \theta_i)$$

From  $N$  observations, personalized modeling methods aim to recover  $N$  parameter estimates  $\hat{\theta}_1, \dots, \hat{\theta}_N$ . Without further assumptions this problem is ill-defined, and the estimators have far too much variance to be useful. We can begin to make this problem tractable by imposing assumptions on the topology of  $\theta$ , or the relationship between  $\theta$  and contextual variables.

### Population Models

The fundamental assumption of most models is that samples are independent and identically distributed. However, if samples are identically distributed they must also have identical parameters. To account for parameter heterogeneity and create more realistic models we must relax this assumption, but the assumption is so fundamental to many methods that alternatives are rarely explored. Additionally, many traditional models may produce a seemingly acceptable fit to their data, even when the underlying model is heterogeneous. Here, we explore the consequences of applying homogeneous modeling approaches to heterogeneous data, and discuss how subtle but meaningful effects are often lost to the strength of the identically distributed assumption.

Failure modes of population models can be identified by their error distributions.

**Mode collapse:** If one population is much larger than another, the other population will be underrepresented in the model.

**Outliers:** Small populations of outliers can have an enormous effect on OLS models in the parameter-averaging regime.

**Phantom Populations:** If several populations are present but equally represented, the optimal traditional model will represent none of these populations.

**Lemma:** A traditional OLS linear model will be the average of heterogeneous models.

## Context-informed models

### Conditional and Cluster Models

While conditional and cluster models are not truly personalized models, the spirit is the same. These models make the assumption that models in a single conditional or cluster group are homogeneous. More commonly this is written as a group of observations being generated by a single model. While the assumption results in fewer than  $N$  models, it allows the use of generic plug-in estimators. Conditional or cluster estimators take the form

$$\hat{\theta}_0, \dots, \hat{\theta}_C = \arg \max_{\theta_0, \dots, \theta_C} \sum_{c \in \mathcal{C}} \ell(X_c; \theta_c)$$

where  $\ell(X; \theta)$  is the log-likelihood of  $\theta$  on  $X$  and  $c$  specifies the covariate group that samples are assigned to, usually by specifying a condition or clustering on covariates thought to affect the distribution of observations. Notably, this method produces fewer than  $N$  distinct models for  $N$  samples and will fail to recover per-sample parameter variation.

### Distance-regularized Models

Distance-regularized models assume that models with similar covariates have similar parameters and encode this assumption as a regularization term.

$$\hat{\theta}_0, \dots, \hat{\theta}_N = \arg \max_{\theta_0, \dots, \theta_N} \sum_i [\ell(x_i; \theta_i)] - \sum_{i,j} \frac{\|\theta_i - \theta_j\|}{D(c_i, c_j)}$$

The second term is a regularizer that penalizes divergence of  $\theta$ 's with similar  $c$ .

### Parametric Varying-coefficient models

Original paper (based on a smoothing spline function): [1] Markov networks: [2] Linear varying-coefficient models assume that parameters vary linearly with covariates, a much stronger assumption than the classic varying-coefficient model but making a conceptual leap that allows us to define a form for the relationship between the parameters and covariates.

$$\hat{\theta}_0, \dots, \hat{\theta}_N = \hat{A}C^T$$

$$\hat{A} = \arg \max_A \sum_i \ell(x_i; Ac_i)$$

### Semi-parametric varying-coefficient Models

Original paper: [3] 2-step estimation with RBF kernels: [4]

Classic varying-coefficient models assume that models with similar covariates have similar parameters, or – more formally – that changes in parameters are smooth over the covariate space. This assumption is encoded as a sample weighting, often using a kernel, where the relevance of a sample to a model is equivalent to its kernel similarity over the covariate space.

$$\hat{\theta}_0, \dots, \hat{\theta}_N = \arg \max_{\theta_0, \dots, \theta_N} \sum_{i,j} \frac{K(c_i, c_j)}{\sum_k K(c_i, c_k)} \ell(x_j; \theta_i)$$

This estimator is the simplest to recover  $N$  unique parameter estimates. However, the assumption here is contradictory to the partition model estimator. When the relationship between covariates and parameters is discontinuous or abrupt, this estimator will fail.

## Contextualized Models

Seminal work [5] Contextualized ML generalization and applications: [6], [7], [8], [9], [10], [11], [12], [13]

Contextualized models make the assumption that parameters are some function of context, but make no assumption on the form of that function. In this regime, we seek to estimate the function often using a deep learner (if we have some differentiable proxy for probability):

$$\hat{f} = \arg \max_{f \in \mathcal{F}} \sum_i \ell(x_i; f(c_i))$$

## Latent-structure Models

### Partition Models

Markov networks: [14] Partition models also assume that parameters can be partitioned into homogeneous groups over the covariate space, but make no assumption about where these partitions occur. This allows the use of information from different groups in estimating a model for a each covariate. Partition model estimators are most often utilized to infer abrupt model changes over time and take the form

$$\hat{\theta}_0, \dots, \hat{\theta}_N = \arg \max_{\theta_0, \dots, \theta_N} \sum_i \ell(x_i; \theta_i) + \sum_{i=2}^N \text{TV}(\theta_i, \theta_{i-1})$$

Where the regularizaiton term might take the form

$$\text{TV}(\theta_i, \theta_{i-1}) = |\theta_i - \theta_{i-1}|$$

This still fails to recover a unique parameter estimate for each sample, but gets closer to the spirit of personalized modeling by putting the model likelihood and partition regularizer in competition to find the optimal partitions.

## Fine-tuned Models and Transfer Learning

Review: [15] Noted in foundational literature for linear varying coefficient models [3]

Estimate a population model, freeze these parameters, and then include a smaller set of personalized parameters to estimate on a smaller subpopulation.

$$\begin{aligned} \hat{\gamma} &= \arg \max_{\gamma} \ell(\gamma; X) \\ \hat{\theta}_c &= \arg \max_{\theta_c} \ell(\theta_c; \hat{\gamma}, X_c) \end{aligned}$$

## Context-informed and Latent-structure models

Seminal paper: [16]

Key idea: negative information sharing. Different models should be pushed apart.

$$\hat{\theta}_0, \dots, \hat{\theta}_N = \arg \max_{\theta_0, \dots, \theta_N, D} \sum_{i=0}^N \prod_{j=0 \text{ s.t. } D(c_i, c_j) < d}^N P(x_j; \theta_i) P(\theta_i; \theta_j)$$

# Theoretical Foundations and Advances in Varying-Coefficient Models

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## Principles of Adaptivity

TODO: Analyzing the core principles that underpin adaptivity in statistical modeling.

## Advances in Varying-Coefficient Models

TODO: Outlining key theoretical and methodological breakthroughs.

## Integration with State-of-the-Art Machine Learning

TODO: Assessing the enhancement of VC models through modern ML technologies (e.g. deep learning, boosted trees, etc).

## Context-Invariant Training

TODO: The converse of VC models, exploring the implications of training context-invariant models. e.g. out-of-distribution generalization, robustness to adversarial attacks.

## Context-Adaptive Interpretations of Context-Invariant Models

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In the previous section, we discussed the importance of context in model parameters. Such context-adaptive models can be learned by explicitly modeling the impact of contextual variables on model parameters, or learned implicitly in a model containing interaction effects between the context and the input features. In this section, we will focus on recent progress in understanding how context influences interpretations of statistical models, even when the model was not originally designed to incorporate context.

TODO: Discussing the implications of context-adaptive interpretations for traditional models. Related work including LIME/DeepLift/DeepSHAP.

## Opportunities for Foundation Models

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### Expanding Frameworks

TODO: Define foundation models, Explore how foundation models are redefining possibilities within statistical models.

### Foundation models as context

TODO: Show recent progress and ongoing directions in using foundation models as context.

## Applications, Case Studies, and Evaluations

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### Implementation Across Sectors

TODO: Detailed examination of context-adaptive models in sectors like healthcare and finance.

## **Performance Evaluation**

TODO: Successes, failures, and comparative analyses of context-adaptive models across applications.

## **Technological and Software Tools**

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### **Survey of Tools**

TODO: Reviewing current technological supports for context-adaptive models.

### **Selection and Usage Guidance**

TODO: Offering practical advice on tool selection and use for optimal outcomes.

## **Future Trends and Predictions**

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### **Emerging Technologies**

TODO: Identifying upcoming technologies and predicting their impact on context-adaptive learning.

### **Advances in Methodologies**

TODO: Speculating on potential future methodological enhancements.

## **Open Problems**

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### **Theoretical Challenges**

TODO: Critically examining unresolved theoretical issues like identifiability, etc.

### **Ethical and Regulatory Considerations**

TODO: Discussing the ethical landscape and regulatory challenges, with focus on benefits of interpretability and regulatability.

### **Complexity in Implementation**

TODO: Addressing obstacles in practical applications and gathering insights from real-world data.

TODO: Other open problems?

## **Conclusion**

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### **Overview of Insights**

TODO: Summarizing the main findings and contributions of this review.

## **Future Directions**

TODO: Discussing potential developments and innovations in context-adaptive statistical inference.

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